Mini-Project Draft Chapter 3 Report

**MA7080 Mathematical Modelling**

**Team: SRB60**

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# **Mini-Project Chapter 3 -Covid 19 Modelling**

Introduction:

To model the covid -19 pandemic, so far, we have used Exponential Model and Logistic Model for prediction in Chapter 1 and SIR model in Chapter 2. Now, for Chapter 3, we are extending the SIR model by introducing some psychological factors of human behaviour as per the classical theory of stress and general adaptation syndrome (GAS) by Hans Selye.

We are continuing our analysis using the below 3 countries:

1. The United Kingdom
2. United States of America
3. Italy

For analysis, we are considering the Cumulative Covid cases from the dataset : [WHO-COVID-19-global-data.csv](https://blackboard.le.ac.uk/bbcswebdav/pid-3142604-dt-content-rid-14290443_2/xid-14290443_2) .

Extended SIR Model

The classic SIR model has three states namely: Susceptible which is number of susceptible individuals, infected which is number of infected individuals and recovered is number of removed (recovered/dead) individuals.

According to the theory of GAS, there are three phases of Stress: Alarm, Resistance, and Exhaustion and therefore, we are introducing four types of human behaviour and four subpopulations in S:

* – “Ignorant people that do not know anything worrying about the epidemic.
* – people in “Alarm phase”.
* – people in “Resistance” state, with very rational and save behaviour.
* –people in “Exhaustion” state. They are tired of the epidemic, behave unsafe and do not

Therefore, we can write,

.

We can distribute the alarm phase partially in and and consider only 3 states as below.

and adding to SIR the transitions would be

By the law of conservation , .

# Task 1

For the Extended SIR model, we can write the stress reactions, their reaction rates, and Stoichiometric vectors as below where are the parameters (reaction rate constants) for each equation.

|  |  |  |
| --- | --- | --- |
| Reactions | Reaction rate | Stoichiometric vector |
|  |  | (-1, 0, 0, 1, 0)T |
|  |  | (-1, 1, 0, 0, 0)T |
|  |  | (0, -1, 1, 0, 0)T |
|  |  | (0, 0, -1, 1, 0)T |
|  |  | (0, 0, 0, -1, 1)T |
|  |  | (1, 0, -1, 0, 0)T |

*Table 1. SIR reactions with reaction rates and respective Stoichiometric vectors*

We can write Kinetic equations as below:

Here, the parameters are - Infection rate, - Stress response rate, - exhaustion rate, b - recovery rate,

We are considering the initial values for the parameters as per out assumptions as below:

* After 50 days in “Resistance” state people become tired/exhausted. So, reaction rate constant for 1/50 = 0.02
* After 100 days in “Exhaustion” state people return to the initial ”Ignorant” state and become sensitive to the alarm signals. So, reaction rate constant for 1/100 = 0.01.
* The proportion of I is close to 1 then ignorant people modify their behaviour to resistant with characteristic time 1 day. So, Reaction rate constant for , 1.

# By taking the Normalized cumulative cases, we can plot the graph for Italy as shown below in Figure 1.

Chart, line chart

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*Figure 1. Normalized Graph for Italy with covid start date as 24/02/2020*

# Task 2

The above equations in task 1 are integrated numerically by using the Initial values of a, b, and by plotting, we get the graph in figure 2 below.

a = 0.12722; b = 0.1;

R0, I0, S0 – Initial values as calculated in Chapter 2.

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*Figure 2. Graph for Italy with Initial values of parameters*

MSE = 9.21E-04

A = 0.1231010193

MSE = 2.07E-04Chart, line chart

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In the above graph of observed normalised data and the predicted data calculated using the integrated equations gives a fair approximation of almost all of the data with reasonable error. The Initial segment or the first wave seems to be fitted well when compared to the second wave. Thus, we can adjust few parameters for better prediction.

Similarities: ??

Differences: ??

Improvements / Modifications:

* Our Initial hypothesis was after 50 days in Resistance state people transition to exhaustion state (i.e., Reaction rate constant for 1/50) .

But when we observe the cumulative fraction graph for Italy , after first wave we can see a horizontal segment in the graph for about 100 days which indicates a plateau of cases. Therefore, we can assume that it could take longer than 50 days to go to exhaustion state from resistance for Italy.

So, we can assume, 1/50

* We assumed that it would take 100 days for people to return to the initial ”Ignorant” state from “Exhaustion” state and become sensitive to the alarm signals (i.e., Reaction rate constant for 1/100).

If we assume that people take more time to reach exhaustive state from resistance , it means they are more resilient and it would be sensible to assume that they would also be quick to go back to ignorant state and be responsive to stress. Hence, we can reduce the number of days for transition is less than 100.

So, we can assume 1/100

* We have considered that Reaction rate constant for is, 1, because we assume when Infected is close to 1 then people in ignorant state change their behaviour to resistant in 1 day time.

Our initial assumption of transmission period is 10 days. Therefore, by the 4th or 5th day they would have already been infected. So, we can assume that could be more than 1 day but not exceeding 4 days.

So, we would keep it unchanged

Even though there is possibility to modify all three constants, for our further predictions we are only modifying , as changes in can be negligible.

# Task 3

Based on our proposed modifications in the constants , we can try with various values and analyse the resultant graphs. Note that we are keeping the value of unchanged.

**Modifying :**

Let us consider for which initial . We proposed to increase the number of days to 100 and we can check with less than 50 to compare the results.

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*Figure 3. Graph for Italy with different values of while keeping the rest constant*

|  |  |
| --- | --- |
| **K3 value** | **Error** |
| 1/50 =0.02 | 2.07E-04 |
| 1/100 =0.01 | 8.21E-05 |
| 1/30 = 0.03 | 7.79E-04 |

*Table 2.* *values and their respective MSE*

From the above graphs in figure 3, K3 = 1/100 gives good approximation with least error (see table 2) for the whole wave whereas for other K3 values the last section of data is not being fitted. Hence, we can modify the K3 value to increase from 50 to 100 i.e., people of Italy take more days to go from resistance to exhaustion state.

**Modifying**

For the reaction rate constant of 1/100, we proposed to reduce the number of days people take to go back to initial state of ignorance to 50. We can cross validate our choice by increasing the initial value by 50. We will check these modifications while using initial and modified values.

Chart, line chart

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*Figure 4. Graph for Italy with different values of initial value(left) and changed value (right)*

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| --- | --- | --- |
| **K6 values** | **Error (Initial K3 =1/50)** | **Error (modified K3 =1/100)** |
| 1/50 | 1.74E-04 | 8.46E-05 |
| 1/100 | 2.07E-04 | 8.21E-05 |
| 1/150 | 2.28E-04 | 8.10E-05 |

*Table 3.*  *values and their respective MSE*

By observing the above graphs in figure 4 and errors in table 3, we can say that for different there is no major change in the prediction curve in both cases of but the mean square error seems to come down very slightly for initial when we consider reducing number of days people take to return to Ignorant state from exhaustion state. This means that, this reaction rate constant does not have significant impact on the

For further analysis, we are proposing to reducing the number of days for going back to ignorance state thus by increasing value.

# Task 4

**The crowd effect**:

The crowd effect can be assumed to take place when the transition rate is proportional to the square of the infected fraction I. This assumption is depicting the alarm reaction to increase super linearly when the number of infected people increases. Previously, we have

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We can write the new reaction and reaction rate with q as new constant as

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We evaluate q assuming that for some selected proportion of infected I the reaction rate is the same as for the linear reaction.

Let us consider , where, characterises the “visibility” of epidemic and this depends on activity of mass-media.

, Then

The new equations with the crowd effect can be written as below:

# Task 5

Chart, line chart

Description automatically generated For Ip = 0.02 ; k2 = 1 or 2 ; k6 = 0.02 ; k3 = 0.01

Chart, line chart

Description automatically generated For Ip = 0.004 ; k2 = 2 ; k6 = 0.02 ; k3 = 0.01

Chart, line chart

Description automatically generated For Ip = 0.002 ; k2 = 1 ; k6 = 0.02 ; k3 = 0.01

Chart, line chart

Description automatically generated For Ip = 0.1 ; k2 = 1 ; k6 = 0.02 ; k3 = 0.01